Subcarrier exclusion techniques for coded OFDM systems

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***Abstract*— In OFDM based wireless LANs an efficient coding scheme, e.g. turbo coding, can improve the performance considerably. However, the bit error rate is mainly affected by strongly attenuated subcarriers. In this paper we propose two algorithms to adapt the transmission to the channel conditions by leaving out weak subcarriers when necessary. This way, the performance is less dependent on the fading channel, though the data rate is degraded slightly. In a first approach the average channel capacity is increased. The second proposal provides a solution that bounds the bit error rate even under very poor channel conditions. The algorithms are derived and verified for an indoor propagation channel at 17 GHz.**

***Index Terms*— subcarrier exclusion, adaptive transmission, information theory, turbo coding, OFDM**

I. Introduction

In an OFDM system the total bandwidth is splitted into several equally spaced subchannels. Each subchannel operates with a particular subcarrier frequency, where the subcarrier frequencies are orthogonal to each other. Since the channel has usually a wideband characteristic, the channel transfer function is frequency selective. Each subchannel can be assumed to be non-frequency selective, if the number of subchannels is large enough for the occupied bandwidth. Thus, every subchannel can be considered as an AWGN channel. Though the average received power is roughly equal, some subcarriers may be significantly attenuated due to deep fades. This results in a high bit error rate (BER) on those subcarriers. Even though most subcarriers may be received without errors, the overall bit error rate is mainly dominated by the few subcarriers with the smallest magnitudes. That means, the performance of the system strongly depends on the channel fading characteristic.

In order to achieve the best performance for a given (or estimated) channel transfer function, transmission parameters have to be selected carefully. There are several proposals to adapt the transmission to the channel conditions. Depending on the channel fading characteristic the allocated power and the modulation scheme for each carrier can be varied. An optimal power allocation is also known as Water-Filling [1]. The aim of this technique is to maximize the channel capacity under the constraint of a certain total power budget. The adaption of the modulation scheme for each carrier is usually calculated by bit loading algorithms [2], [3]. As in wireless networks the channel fading characteristic can change quickly, using these techniques results in a large signaling effort. Thus, an amount of the gained performance can get lost.

As simplification, some subcarriers can be switched off, instead of using an adaptive modulation and/or adaptive power scheme. This way, the signaling effort can be strongly reduced. Some basics of this technique were investigated in [4]. There, the idea was to leave out a constant number of subcarriers, where always the weakest carriers are selected. This method allows only little adaption, since the number of excluded subcarriers is fixed. In this paper we propose techniques that select the number of excluded subcarriers adaptively. Thus, the number of used subcarriers is reduced only for bad channel conditions and in many cases all subcarriers are used and no bandwidth is wasted.

This paper is organized as follows. Section II gives a brief description of the employed system model. In section III we analyze the effects of the fading channel on the system performance. Two different schemes for adaptive subcarrier exclusion are derived in section IV. The derived techniques are analyzed and compared with respect to bit error rate performance and data rate degradation in section V. Section VI concludes the paper.

II. System model

The considered OFDM system employs $N = 128$ subcarriers in a 50 MHz channel. Standard cyclic prefix (CP) OFDM is used with a guard interval long enough to prevent inter-symbol interference. Perfect synchronization in time and frequency is assumed. Consequently, the relationship between the transmitted subcarrier symbol X_k and the received subcarrier symbol Y_k can be described as

$$Y_k = H_k X_k + V_k, \tag{1}$$

where H_k denotes the complex fading coefficient of the k^{th} subcarrier and V_k the Gaussian noise transformed into frequency domain.

For the channel coding a turbo code with variable code rate achievable by puncturing has been selected. In the encoder the code polynomials $(13, 15)_{octal}$ are used to generate the redundant bits. Here, we use a block length of 1024 bit. The decoder is based on a Max-LogMAP algorithm, where four iterations are carried out. Soft decision decoding is performed.

We use a channel model for an indoor environment at 17 GHz. This model has a non line of sight (NLOS) characteristic and considers 17 paths. A complete description of the channel is given in [5].

III. Fading channel effects

Usually, in OFDM systems we have to deal with wideband frequency selective channels. It is well known that the performance decreases for those fading channels compared to flat AWGN channels, although the mean power gain of both channels is rather equal. That means, the performance highly depends on the frequency selectivity of the channel. Even in a particular environment (e.g. an office) an infinite number of channel realizations with different fading characteristics can occur. In order to ensure transmission at a target BER, it is important to assess the channel conditions. Thus, we have to find a measure to indicate the frequency selectivity of the channel. We define the average logarithmic attenuation (ALA)

$$D = -\frac{1}{N}\sum_{k=0}^{N-1} 10 \cdot \log(|H_k|^2) \text{ [dB]}. \tag{2}$$

An ALA equal to zero means the whole channel has a flat fading characteristic, i.e. the channel is an AWGN channel. The higher the ALA the larger is the performance degradation of the system. In figure 1 the transfer functions of two channel realizations are plotted. Note, the overall gain of the two realizations is normalized to one. The ALA of one realization is close to zero, which means the system performance is close to the performance of an AWGN system. The ALA of the second realization is much higher, which corresponds to a large performance degradation.

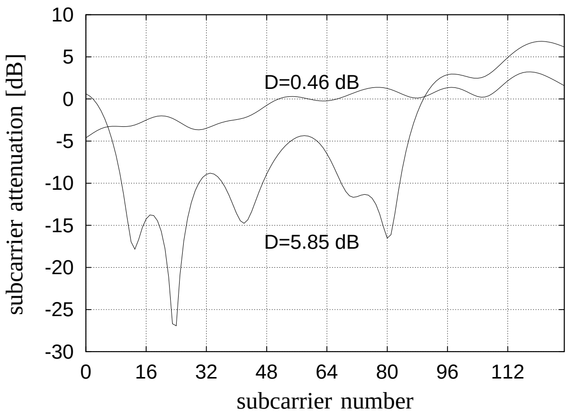

Fig. 1. Channel realizations with different ALA

In order to show the dependence between the system performance and the ALA of the according channel transfer function, we subdivide all possible channel realizations into ten groups.[1] The first nine groups contain the channel realizations with an ALA from 0 to 4.5 dB. The limits are equally spaced by 0.5 dB. The channel realizations with an ALA larger than 4.5 dB form the 10^{th} group. Figure 2 shows the probability that an arbitrary channel realization belongs to one of the ten groups.

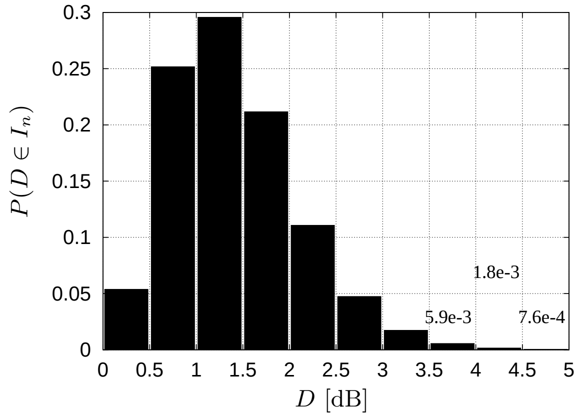

Fig. 2. Probabilities of the groups of channel realizations

The BER performance corresponding to the defined groups is presented in figure 3. In addition the BER for an AWGN channel and the average BER for the investigated channel model are plotted as dashed lines. It can easily be seen that the BER performance loss is proportional to the ALA. Furthermore, at a BER of 10^{-5} the required E_b/N_0 can vary from about -4 dB to more than $+4$ dB compared to the average required E_b/N_0.

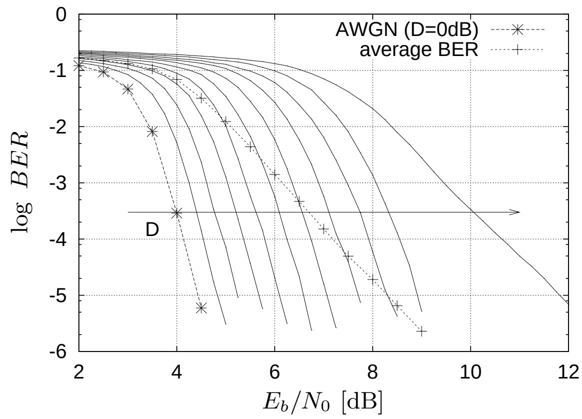

Fig. 3. BER for channels groups with different ALA, 16-QAM, code rate 1/2

The relationship between the average BER ($\overline{BER}$) and the BER for the channel realization groups ($BER(I_n)$) can be calculated with the Bayes' theorem

$$\overline{BER} = \sum_n BER(I_n) \cdot P(D \in I_n). \tag{3}$$

According to figure 3 we can assume for a small average BER: $BER(I_n|n = 1..9) \ll BER(I_{10})$. For this condition we can approximate equation 3

$$\overline{BER} \approx BER(I_{10}) \cdot P(D \in I_{10}). \tag{4}$$

That means, for a small average BER the BER mainly depends on the BER for the worst channel realizations. Equation 4 can be easily verified in figure 3. The graphs for the average BER and the BER for the worst channel realizations are nearly parallel and differ by a factor of about $P(D \in I_{10})$ for lower BER.

[1] Of course it would be the best to choose an infinite number of channel groups, i.e. each channel realization forms one group. This causes a very large effort in simulation and visualization. As a trade-off between simulation effort and realistic considerations we chose ten groups. So, we assume that all channel realizations of one group have a similar behavior with respect to system performance and ALA.

From these considerations we can conclude: when selecting the transmission parameters, it is not optimal to focus on the average performance of the system. Since the performance can vary significantly, in many cases a large amount of power is wasted or the achieved BER is very poor. In order to allow efficient transmission, parameters have to be adapted to the channel conditions.

IV. Subcarrier exclusion techniques

A. Maximizing the average channel capacity

Since a powerful error correction coding allows transmission near the Shannon limit, it is useful to consider the channel capacity as a measure for the system performance. The channel capacity is an upper bound for the data rate of a communication system.

The capacity of one subchannel can be calculated by the theorems of Shannon [6]. In an OFDM system the overall capacity is the sum of all subchannel capacities. Since the carrier attenuations are time-variant, the overall channel capacity is time-variant, too. Thus, it is common to indicate the average channel capacity for an infinite number of OFDM symbols. If the probability density function (PDF) of the channel coefficients $p(H)$ is known, the average subchannel capacity is given by

$$C_{\text{Fading}}\left(\frac{E_s}{N_0}\right) = \int\limits_0^\infty C_{\text{AWGN}}\left(|H|^2\frac{E_s}{N_0}\right)p(H)dH, \quad (5)$$

where $C_{\text{AWGN}}(.)$ is the capacity of one single AWGN channel and $C_{\text{Fading}}(.)$ is the average capacity. The ratio E_s/N_0 denotes the SNR of one subcarrier symbol. According to equation 5 the channel capacity is given in $bits/symbol$.

One technique to increase the channel capacity is the well known Water-Filling. The goal of this method is to maximize the channel capacity for a given total power budget. For each carrier an optimal power is determined according to its attenuation. If the attenuation of several subcarriers is too high, the assigned power is zero. That means, those subcarriers are not used to carry information. From this algorithm a carrier exclusion scheme can be derived. Then, the subcarriers whose assigned power is zero are left out and the total power is distributed equally over the remaining subcarriers. This solution contains some significant disadvantages:

- it is valid only for a continuous Gaussian modulation scheme, rather than for discrete input signal sets like M-QAM
- it is not proofed, that the number of excluded subcarriers is optimal to maximize the channel capacity
- complex calculations are necessary to select the subcarriers to be excluded.

Now, an algorithm will be derived that considers the disadvantages mentioned above. In the following, we call this algorithm CMSE (Capacity Maximizing Subcarrier Exclusion). We define the average portion of used subcarriers α as the average ratio of the number of used subcarriers to the total number of subcarriers. If α is a degree of freedom, there is an optimal value that maximizes the average channel capacity for a certain modulation scheme. This way, the average channel capacity is a function of the average SNR and α

$$C^{SS}_{\text{Fading}}\left(\alpha, \frac{\overline{E}_s}{N_0}\right) = \int\limits_{H_T}^\infty C^{SS}_{\text{AWGN}}\left(\frac{|H|^2}{\alpha}\frac{\overline{E}_s}{N_0}\right)p(H)dH, \quad (6)$$

where the superscript means the channel capacity is calculated for a certain input signal set, which is different from the Water-Filling solution. The average assigned energy $\overline{E}_s$ corresponds to the allocated energy per carrier when all subcarriers are used. The integration limit H_T is a threshold for the channel coefficients. If a channel coefficient is smaller than this threshold, the according carrier will be excluded. The relationship between the threshold and the average portion of used subcarriers is given by

$$\alpha = \int\limits_{H_T}^\infty p(H)dH. \quad (7)$$

The maximum of the average channel capacity is defined by

$$C^{SS}_{\text{Fading}}\left(\frac{\overline{E}_s}{N_0}\right) = \max_\alpha \left[C^{SS}_{\text{Fading}}\left(\alpha, \frac{\overline{E}_s}{N_0}\right)\right]. \quad (8)$$

Usually, this can only be solved by numerical evaluation, since the capacity formula for an arbitrary signal set has high complexity. Besides, the PDF of the channel coefficients is usually given as a numerical estimation.

The maximal average channel capacity for several modulation schemes is presented in figure 4. It can be concluded

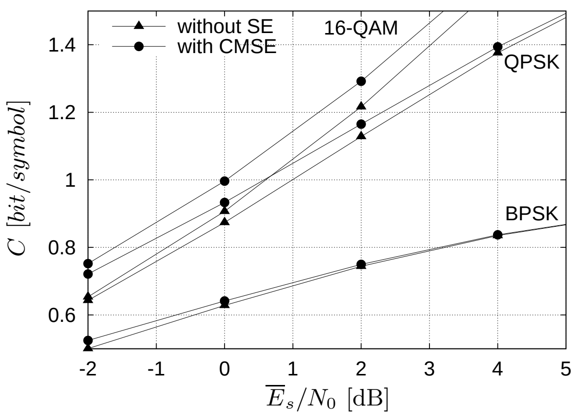

Fig. 4. Average channel capacity with CMSE

that a significant gain is only possible for low SNR. That means, if the SNR increases to a certain value, almost no subcarriers will be excluded. In figure 5 the average portion of used subcarriers is shown. Although the rate of excluded subcarriers is relatively small for typical SNR values, the portion can considerably vary for single channel realizations. In subsection V we show the dependence between the ALA of a channel transfer function and the portion of excluded carriers.

ular scenario a trade-off between data rate reduction and performance gain has to be made.

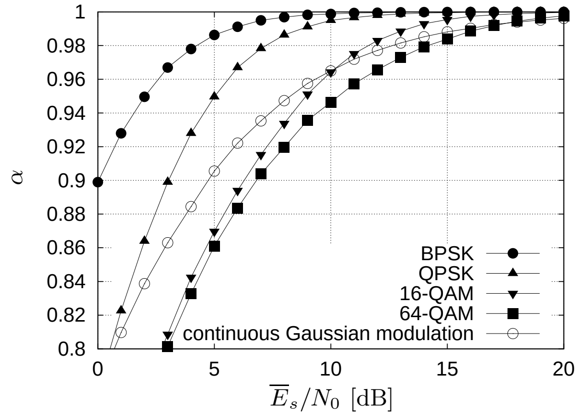

Fig. 5. Average portion of used subcarriers (CMSE)

B. Bounding the bit error rate

The nature of the technique investigated above is rather theoretical, since the channel capacity as an upper bound is only reachable by using an ideal coding scheme and an infinite time diversity. From the practical point of view, the achievable bit error rate is more interesting than the channel capacity. The BER performance does not only depend on the channel capacity, but also on the particular error correction code. In order to focus more on the BER performance, the coding impact on the bit error rate has to be considered.

As mentioned in section III, the performance for a particular channel realization can be predicted roughly by calculating the average logarithmic attenuation. Assuming the bit error rate increases for larger ALA, a bounding of the ALA leads to a bounding of the bit error rate. For this reason we define a cut-off value $D_{\rm co}$ that bounds the ALA. If the ALA of a particular channel realization is equal to or smaller than the cut-off value, all subcarriers are used to carry information. In the other case the weakest subcarriers are selected and left out until the ALA is equal to or smaller than the cut-off value. Then, only the used subcarriers are considered to recalculate the ALA. We name this technique BBSE (Bit error rate Bounding Subcarrier Exclusion).

The resulting average portion of used subcarriers is plotted over the cut-off value in figure 6. Note, one can vary

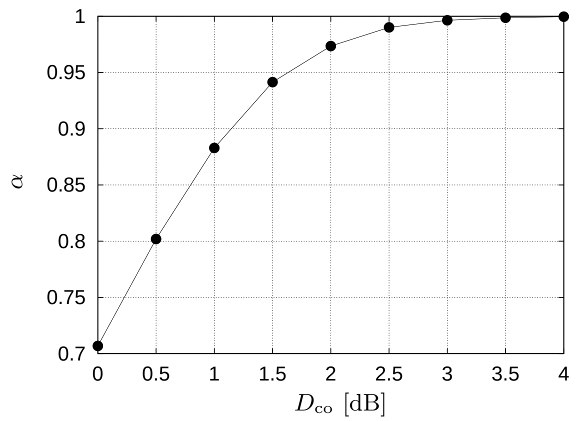

Fig. 6. Average portion of used subcarriers (BBSE)

the average portion of excluded subcarriers by adjusting the cut-off value. When the cut-off value decreases, the performance is expected to improve. Hence, for a partic-

V. Simulated performance and comparisons

In the previous section two different carrier exclusion schemes have been derived. In the following, the BER performance of these techniques based on simulations is presented.

All simulations have been carried out under the following conditions. For each data block of 1024 bit an independent channel realization is generated. The overall channel gain is normalized to one for each channel realization. During the transmission of one data block the channel is supposed to be stationary. For all simulations we employ 16-QAM and a code rate of 1/2. Similar results can be achieved with other modulation schemes (BPSK, QPSK, 64-QAM) and code rates (1/3, 3/4).

In figure 7 the average bit error rate is presented. The performance for BBSE is shown for different cut-off values. If the derived carrier exclusion schemes are applied,

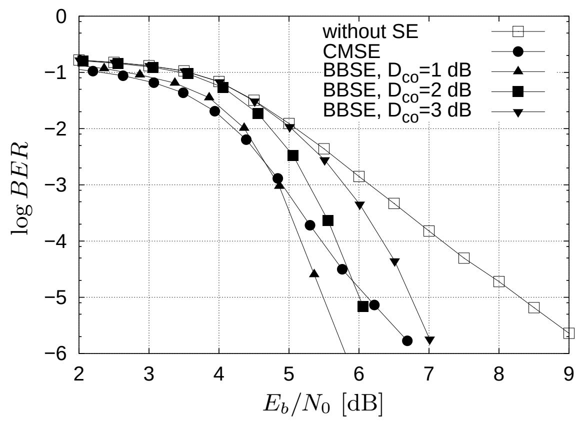

Fig. 7. Average BER using subcarrier exclusion

the average BER improves significantly. While the BER for CMSE is fixed, the BER for BBSE highly depends on the cut-off value. If a smaller cut-off value is chosen, the performance gets better and the throughput decreases. So, for each application a good compromise between BER performance and data rate reduction has to be found.

In order to compare the two schemes, the portion of excluded subcarriers has to be considered. In Table I the average portion of excluded subcarriers is summarized for a BER of 10^{-5}. Taking the BER performance and the data

TABLE I

Average portion of excluded subcarriers at a BER of 10^{-5}

	CMSE	BBSE		
D_{co}		1 dB	2 dB	3 dB
$1-\alpha$	4.9%	11.7%	2.5%	0.4%

rate reduction into account, the benefits are clearly on the side of BBSE.

As we have seen in section III, the BER for a particular channel realization can differ significantly from the average BER. If we focus on the average BER performance (see figure 7), for one point, the BER is averaged for all channel realizations at one particular SNR. In this case, the BER for good channel conditions can be more than sufficient, while the BER for bad channel conditions can be very poor (see figure 3). In order to allow efficient transmission, it is always desired to have a BER close to a target BER, no matter of the actual channel conditions. Thus, it is more interesting to focus on the performance gain for particular channel realizations at a target BER, rather than for an infinite number of channel realizations at a fixed SNR. Therefore, simulations were done with the channel realization groups, defined in section III. For each of the ten groups the required E_b/N_0 to reach a target BER of 10^{-3} and 10^{-5} is plotted in figure 8 and 9, respectively.

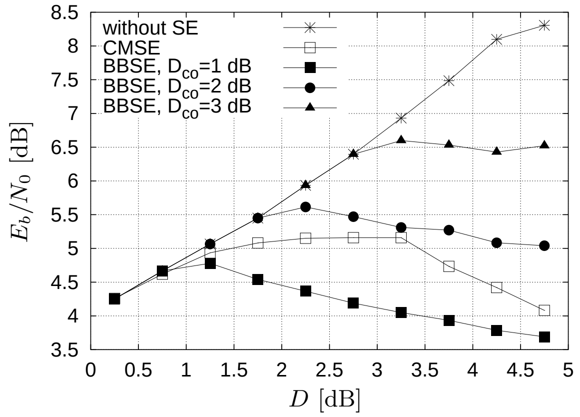

Fig. 8. Required E_b/N_0 to reach a target BER of 10^{-3} for the channel groups

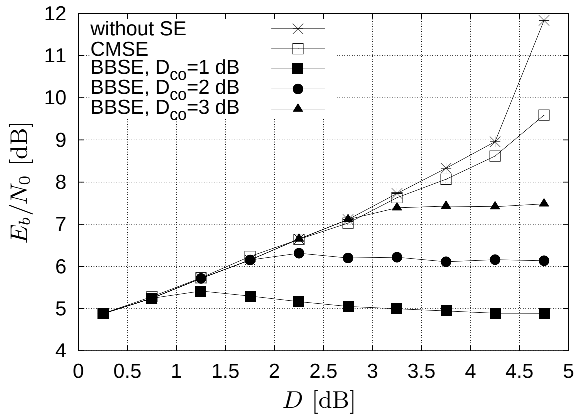

Fig. 9. Required E_b/N_0 to reach a target BER of 10^{-5} for the channel groups

For CMSE the performance gain at a BER of 10^{-3} is quite high, while the gain is almost vanished at a BER of 10^{-5}. That means, the BER performance improves only for small BER. It seems that this is a contradiction to the average BER (see figure 7), where the performance gain is about 2.5 dB for a BER of 10^{-5}. In fact, this gain comes from the bad channel realizations, which cause a high BER at the simulated E_b/N_0. So, it is essential to assess the performance gain for particular channel realizations (or at least channel groups with similar behavior), rather than for an infinite number of channel realizations.

When BBSE is applied, the BER fluctuation can be reduced very effectively. While the BER for channel realizations with an ALA less than the cut-off value remains unaffected, the BER for worse channel realizations is stabilized. That means, one can set an upper bound for the BER by selecting an appropriate cut-off value.

In figure 10 the portion of excluded subcarriers for each channel group is presented. Using BBSE the portion of excluded subcarriers only depends on the cut-off value. For CMSE two graphs are plotted, since the data rate depends on the SNR (and hence also on the BER). If we look at a BER of 10^{-5}, one can see that the portion of excluded subcarriers can be smaller for BBSE, although the performance is better compared to CMSE. That means, the BBSE scheme is clearly more effective than CMSE.

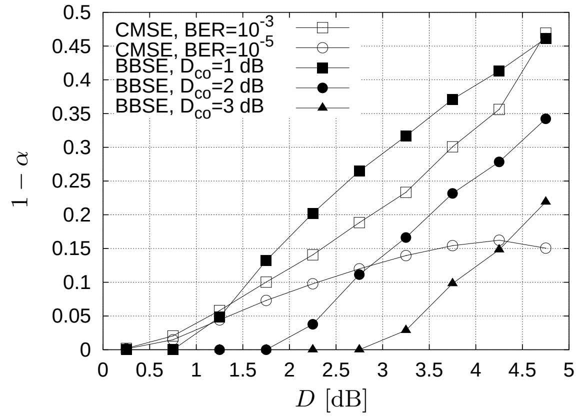

Fig. 10. Portion of excluded subcarriers for the channel groups

VI. Conclusion

In this paper two different techniques for subcarrier exclusion have been presented. Considering the diversity properties of an error correction coding, the problem of subcarrier exclusion must be approached in a completely different way than for uncoded systems. At first the channel capacity, as an upper bound for the system performance, was maximized by leaving out weak subcarriers. The aim of the second scheme was to keep the bit error rate bounded, even under very poor channel conditions. The two algorithms were compared with respect to BER performance and data rate degradation. Also, some investigations on correct simulation of carrier exclusion schemes from the practical point of view have been made. While the first scheme is rather theoretical, the second scheme allows a nearly arbitrary reduction of the BER fluctuation by selecting an appropriate cut-off value.

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